DAY THIRTY SEVEN

Unit Test 6

(Statistics, Probability & Mathematical Reasoning)

1 While shuffling a pack of playing cards, four are accidently dropped. The probability that the cards are dropped one from each suit is

(a) $\frac{1}{256}$ (b) $\frac{2197}{20825}$ (c) $\frac{3}{20825}$ (d) None of these

2 If p : Ajay works hard, q : Ajay gets good marks, then proposition $\sim p \Rightarrow \sim q$ is equivalent to

(a) Ajay does not work hard and yet he gets good marks
(b) Ajay work hard if and only if he gets good marks
(c) If Ajay does not work hard, then he does not get good marks
(d) None of the above

3 $\sim [(p \vee q) \wedge \sim (p \wedge q)]$ is equivalent to

(a) $p \Leftrightarrow q$ (b) $\sim p \wedge q$
(c) $\sim (p \Leftrightarrow q)$ (d) None of these

4 Five-digit numbers are formed using the digits 0, 2, 4, 6, 8 without repeating the digits. If a number so formed is chosen at random, probability that it is divisible by 20 is

(a) 1/2 (b) 1/3 (c) 1/4 (d) 2/5

5 For a frequency distribution consisting of 18 observations, the mean and the standard deviation were found to be 7 and 4, respectively. But on comparison with the original data, it was found that a figure 12 was miscopied as 21 in calculations. The correct mean and standard deviation are

(a) 6.7, 2.7 (b) 6.5, 2.5
(c) 6.34, 2.34 (d) None of these

6 A boy is throwing stones at a target. The probability of hitting the target at any trial is 1/2. The probability of hitting the target 5th time at the 10th throw is

(a) $\frac{5}{2^{10}}$ (b) $\frac{63}{2^9}$ (c) $\frac{{}^{10}C_5}{2^{10}}$ (d) $\frac{{}^{10}C_4}{2^{10}}$

7 An automobile driver travels from plane to a hill station, 120 km distant at an average speed of 30 km/h. Then, he makes the return trip at an average speed of 25 km/h. He covers another 120 km distance on plane at an average speed of 50 km/h. His average speed over the entire distance of 360 km will be

(a) $\frac{30+25+50}{3}$ km/h (b) $\frac{25+35+15}{3}$ km/h
(c) $\frac{3}{\frac{1}{30}+\frac{1}{25}+\frac{1}{50}}$ km/h (d) None of these

8 The marks obtained by 60 students in a certain test are given below

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of Students	2	3	4	5	6	12	14	10	4

Find the median of the above data.

(a) 68.33 (b) 70
(c) 71.11 (d) None of these

9 Assuming $(p \vee q)$ is true and $(p \wedge q)$ is false, state which of the following proposition have true values?

(a) $\sim p \wedge q$ (b) $\sim p \vee \sim q$ (c) $p \Leftrightarrow q$ (d) None of these

10 Let S be the universal set and $n(X) = k$. The probability of selecting two subsets A and B of the set X such that $B = \overline{A}$, is

(a) $\frac{1}{2}$ (b) $\frac{1}{2^k - 1}$ (c) $\frac{1}{2^k}$ (d) $\frac{1}{3^k}$

11 The probability that when 12 balls are distributed among three boxes, the first box will contain three balls, is

(a) $\frac{2^9}{3^{12}}$ (b) $\frac{{}^{12}C_3 \cdot 2^9}{3^{12}}$ (c) $\frac{{}^{12}C_3 \cdot 2^{12}}{3^{12}}$ (d) None

12 A fair coin is tossed 100 times. The probability of getting tails an odd number of times is

(a) $\frac{1}{2}$ (b) $\frac{1}{8}$ (c) $\frac{3}{8}$ (d) None of these

13 One mapping is selected at random from all the mappings of the set $A = \{1, 2, 3, \ldots, n\}$ into itself. The probability that the mapping selected is one to one is given by

(a) $\frac{1}{n^n}$ (b) $\frac{1}{n!}$

(c) $\frac{(n-1)!}{n^{n-1}}$ (d) None of these

14 A natural number is selected at random from the set $X = \{x : 1 \le x \le 100\}$. The probability that the number satisfies the inequation $x^2 - 13x \le 30$ is

(a) $\frac{9}{50}$ (b) $\frac{3}{20}$ (c) $\frac{2}{11}$ (d) None of these

15 If $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)P(B)$, then

(a) $P\left(\frac{A}{B}\right) = 0$ (b) $P\left(\frac{B}{A}\right) = 0$

(c) $P(A' \cap B') = P(A')P(B')$ (d) $P(A/B) + P(B/A) = 1$

16 A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is a six. The probability that it is actually a six is

(a) $\frac{3}{8}$ (b) $\frac{1}{5}$ (c) $\frac{3}{4}$ (d) None of these

17 A letter is taken at random from the letters of the word 'STATISTICS' and another letter is taken at random from the letters of the word 'ASSISTANT'. The probability that they are the same letters is

(a) $\frac{1}{45}$ (b) $\frac{13}{90}$ (c) $\frac{19}{90}$ (d) $\frac{5}{18}$

18 An ellipse of eccentricity $\frac{2\sqrt{2}}{3}$ is inscribed in a circle and a point within the circle is chosen at random. The probability that this point lies outside the ellipse is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{9}$ (d) $\frac{2}{9}$

19 Four-digit numbers are formed using each of the digit $1, 2, \ldots, 8$ only once. One number from them is picked up at random. The probability that the selected number contains unity is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) None of these

20 The mean of 10 numbers is 12.5, the mean of the first six is 15 and the last five is 10. The sixth number is

(a) 12 (b) 15

(c) 18 (d) None of these

21 The geometric mean of numbers $7, 7^2, 7^3, \ldots, 7^n$ is

(a) $7^{7/n}$ (b) $7^{n/7}$

(c) $7^{(n-1)/2}$ (d) $7^{(n+1)/2}$

22 The probability that the 14th day of a randomly chosen month is a Saturday, is

(a) $\frac{1}{12}$ (b) $\frac{1}{7}$

(c) $\frac{1}{84}$ (d) None of these

23 The mean of the numbers

$$\frac{{}^{30}C_0}{1}, \frac{{}^{30}C_2}{3}, \frac{{}^{30}C_4}{5}, \ldots\ldots, \frac{{}^{30}C_{30}}{31} \text{ equals to}$$

(a) $\frac{2^{30}}{31}$ (b) $\frac{2^{26}}{31}$

(c) $\frac{2^{26}}{31 \times 15}$ (d) None of these

24 If the mean of a binomial distribution is 25, then its standard deviation lies in the interval

(a) [0, 5) (b) (0, 5]

(c) [0, 25) (d) (0, 25)

25 Let p : She is intelligent and q : She is studious. The symbolic form of "it is not true that she is not intelligent or she is not studious" is

(a) $p \wedge \sim q$ (b) $\sim p \wedge q$

(c) $p \wedge q$ (d) None of these

26 The negation of $p \to (\sim p \vee q)$ is

(a) $p \vee (p \vee \sim q)$ (b) $p \to \sim (p \vee q)$

(c) $p \to q$ (d) $p \wedge \sim q$

27 The proposition of $(p \vee r) \wedge (q \vee r)$ is equivalent to

(a) $(p \wedge q) \vee r$ (b) $(p \vee q) \wedge r$

(c) $p \wedge (q \vee r)$ (d) $p \vee (q \wedge r)$

28 Which of the following statement has the truth value 'F' ?

(a) A quadratic equation has always a real root.

(b) The number of ways of seating 2 persons in two chairs out of n persons is $P(n, 2)$.

(c) The cube roots of unity are in GP.

(d) None of the above

29 The variates x and u are related by $hu = x - a$, then correct relation between σ_x and σ_u is

(a) $\sigma_x = h\sigma_u$ (b) $\sigma_u = h\sigma_x$

(c) $\sigma_x = a + h\sigma_u$ (d) $\sigma_u = a + h\sigma_x$

30 Given that, $x \in [0, 1]$ and $y \in [0, 1]$. If A is the event of (x, y) satisfying $y^2 \le x$ and B is the event of (x, y) satisfying $x^2 \le y$. Then,

(a) $P(A \cap B) = \frac{1}{3}$

(b) A, B are exhaustive

(c) A, B are mutually exclusive

(d) A, B are independent

31 There are two independent events A and B. The probability that both A and B occurs is $\frac{1}{8}$ and the probability that neither of them occurs is $\frac{1}{4}$. Then, the probability of the two events are, respectively

(a) $\frac{7\pm\sqrt{17}}{16}, \frac{2}{7\pm\sqrt{17}}$ (b) $\frac{5\pm\sqrt{14}}{16}, \frac{3}{5\pm\sqrt{14}}$

(c) $\left(\frac{1}{3},\frac{1}{2}\right),\left(\frac{1}{2},\frac{1}{3}\right)$ (d) None of these

32 If $p \Rightarrow (q \vee r)$ is false, then the truth values of p, q, r are respectively.

(a) T, F, F (b) F, F, F (c) F, T, T (d) T, T, F

33 In a factory, workers work in three shifts say shift A, shift B and shift C and they get wages in the ratio 4 : 5 : 6 depending on the shift A, B and C, respectively. Number of workers in the shifts are in the ratio 3: 2 :1. If total number of workers is 1500 and wages per worker in shift A is ₹ 400. The mean wage of a worker is

(a) ₹ 467 (b) ₹500 (c) ₹600 (d) ₹ 400

34 The odd in favour of standing first of three students appearing in an examination are 1:2, 2:5 and 1:7, respectively. The probability that either of them will stand first, is

(a) $\frac{125}{168}$ (b) $\frac{75}{168}$ (c) $\frac{32}{168}$ (d) $\frac{4}{168}$

35 Median of ${}^{2n}C_0, {}^{2n}C_1, {}^{2n}C_2, \ldots\ldots, {}^{2n}C_n$ (when n is odd) is

(a) $\frac{1}{2}\left({}^{2n}C_{\frac{(n-1)}{2}} + {}^{2n}C_{\frac{(n+1)}{2}}\right)$

(b) ${}^{2n}C_{n/2}$

(c) ${}^{2n}C_n$

(d) None of the above

36 The contrapositive of $(p \vee q) \rightarrow r$ is

(a) $\sim r \rightarrow (p \vee q)$ (b) $r \rightarrow (p \vee q)$

(c) $\sim r \rightarrow (\sim p \wedge \sim q)$ (d) $p \rightarrow (q \vee r)$

Direction (Q. Nos. 37-40) *Each of these questions contains two statements : Statement* I *(Assertion) and Statement* II *(Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes* (a), (b), (c) *and* (d) *given below.*

(a) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I

(b) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I

(c) Statement I is true; Statement II is false

(d) Statement I is false; Statement II is true

37 Suppose two groups of scores A and B are such that $A = (x, x + 2, x + 4)$ and $B = (x - 2, x + 2, x + 6)$

Statement I Group B has more variability than group A.

Statement II The value of mean for group B is more than that of group A.

38 Let p : He is poor. q : He is happy.

Statement I The symbolic form of the statement "It is not true that if he is poor, then he is happy" is $\sim (p \Rightarrow q)$.

Statement II The negation of the above statement is $(p \Rightarrow \sim q)$.

39 Let A and B be two independent events.

Statement I If $P(A) = 0.3$ and $P(A \cup \overline{B}) = 0.8$, then $P(B)$ is $\frac{2}{7}$.

Statement II $P(\overline{E}) = 1 - P(E)$, where E is any event.

40 **Statement I** The statements $(p \vee q) \wedge \sim p$ and $\sim p \wedge q$ are logically equivalent.

Statement II The end columns of the truth table of both statements are identical.

ANSWERS

1 (b)	**2** (c)	**3** (a)	**4** (c)	**5** (b)	**6** (b)	**7** (c)	**8** (a)	**9** (b)	**10** (b)
11 (b)	**12** (a)	**13** (c)	**14** (b)	**15** (c)	**16** (a)	**17** (c)	**18** (b)	**19** (a)	**20** (b)
21 (d)	**22** (c)	**23** (b)	**24** (a)	**25** (c)	**26** (d)	**27** (a)	**28** (a)	**29** (a)	**30** (a)
31 (a)	**32** (a)	**33** (a)	**34** (a)	**35** (a)	**36** (c)	**37** (c)	**38** (c)	**39** (b)	**40** (a)

Hints and Explanations

1 Required probability $= \dfrac{13^4}{^{52}C_4} = \dfrac{2197}{20825}$

2 Given, proposition is equivalent to "If Ajay does not work hard, then he does not get good marks".

3 It is clear from the table that column IIIrd and IVth are identical.

p	q	$p \Leftrightarrow q$	$\sim [(p \vee q) \wedge \sim (p \wedge q)]$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

4 The total number of numbers formed is $5! - 4! = 96$.
If a number is divisible by 20, then the last digit is 0 and tens digit must be even. The number of such numbers with zero at the end is $4! = 24$.
$\therefore$ Required probability $= \dfrac{24}{96} = \dfrac{1}{4}$

5 Given, $\dfrac{\Sigma x_i}{18} = 7$
$\Rightarrow \Sigma x_i = 126$
Now, correct $\Sigma x_i = 126 - 21 + 12 = 117$
$\therefore$ True mean $= \dfrac{\Sigma x_i}{18} = \dfrac{117}{18} = 6.5$
Since, $\dfrac{\Sigma x_i^2}{18} - (\text{Mean})^2 = 4^2$
$\therefore \dfrac{\Sigma x_i^2}{18} = 4^2 + (7)^2$
$\Rightarrow \Sigma x_i^2 = 1170$
Now, correct $\Sigma x_i^2 = 1170 - 21^2 + 12^2 = 873$
$\therefore$ True variance
$= \dfrac{\Sigma x_i^2}{18} - (\text{Mean})^2$
$= \dfrac{873}{18} - (6.5)^2$
$= 48.5 - 42.25 = 6.25$
$\therefore$ True standard deviation
$= \sqrt{\text{True variance}}$
$= \sqrt{6.25} = 2.5$

6 Here, $p = \dfrac{1}{2}$ and $q = \dfrac{1}{2}$
$\therefore$ Required probability
$= {^9C_4} \left(\dfrac{1}{2}\right)^4 \left(\dfrac{1}{2}\right)^5 \times \dfrac{1}{2}$
$= {^9C_4} \left(\dfrac{1}{2}\right)^{10}$
$= \dfrac{9 \times 8 \times 7 \times 6}{(1 \times 2 \times 3 \times 4) \times 2^{10}} = \dfrac{63}{2^9}$

7 Average speed $= \dfrac{120 + 120 + 120}{\dfrac{120}{30} + \dfrac{120}{25} + \dfrac{120}{50}}$
$= \dfrac{3}{\dfrac{1}{30} + \dfrac{1}{25} + \dfrac{1}{50}}$ km/h

8

Class	f_i	cf
10-20	2	2
20-30	3	5
30-40	4	9
40-50	5	14
50-60	6	20
60-70	12	32
70-80	14	46
80-90	10	56
90-100	4	60

Here, $N = 60 \Rightarrow \dfrac{N}{2} = 30$
$\Rightarrow$ median class is 60-70.
$\therefore$ Median $= l + \dfrac{\dfrac{N}{2} - c}{f} \times h$
$= 60 + \dfrac{30 - 20}{12} \times 10$
$= 60 + \dfrac{100}{12}$
$= 68.33$

9

p	q	$p \vee q$	$p \wedge q$	$\sim p \wedge q$	$\sim p \vee \sim q$	$p \Leftrightarrow q$
T	F	T	F	F	T	F
F	T	T	F	T	T	F

10 Total number of subsets of X is 2^k.
$\therefore$ Total number of possible out comes
$= {^{2^k}C_2}$
Let $n(E)$ = The number of selections of two non-intersecting subsets whose union is X.
$= \dfrac{1}{2}({^kC_0} + {^kC_1} + {^kC_2} + \ldots)$
$= \dfrac{1}{2} \times 2^k$
$\therefore$ Required probability
$= \dfrac{\dfrac{1}{2} \times 2^k}{^{2^k}C_2} = \dfrac{2^{k-1}}{2^k\left(\dfrac{2^k - 1}{2}\right)}$
$= \dfrac{1}{2^k - 1}$

11 Since, each ball can be put into any one of the three boxes, so that total number of ways in which 12 balls can be put into three boxes is 3^{12}.
The three balls can be chosen in $^{12}C_3$ ways and remaining 9 balls can be put in the remaining 2 boxes in 2^9 ways.
$\therefore$ Required probability
$= \dfrac{^{12}C_3 \cdot 2^9}{3^{12}}$

12 $P(X = \text{odd number})$
$= P(X = 1) + P(X = 3) + \ldots + P(X = 99)$
$= {^{100}C_1}\left(\dfrac{1}{2}\right)^{100} + {^{100}C_3}\left(\dfrac{1}{2}\right)^{100} + \ldots + {^{100}C_{99}}\left(\dfrac{1}{2}\right)^{100}$
$= ({^{100}C_1} + {^{100}C_3} + \ldots + {^{100}C_{99}})\left(\dfrac{1}{2}\right)^{100}$
$= 2^{99} \cdot \left(\dfrac{1}{2}\right)^{100} = \dfrac{1}{2}$

13 Total number of cases $= n^n$
$\therefore$ The number of favourable cases
$= n(n-1)\ldots 2 \cdot 1 = n!$
$\therefore$ Required probability
$= \dfrac{n!}{n^n} = \dfrac{(n-1)!}{n^{n-1}}$

14 Total number of ways = 100
Given, $x^2 - 13x \le 30$
$\Rightarrow \left(x - \dfrac{13}{2}\right)^2 \le \dfrac{289}{4}$
$\Rightarrow -\dfrac{17}{2} \le x - \dfrac{13}{2} \le \dfrac{17}{2}$
$\Rightarrow -2 \le x \le 15$
$\because x \in N$
$\therefore x \in \{1, 2, 3, \ldots, 15\}$
$\therefore$ Required probability
$= \dfrac{15}{100} = \dfrac{3}{20}$

15 Given,
$P(A \cup B) = P(A) + P(B) - P(A)P(B)$
$\therefore P(A \cap B) = P(A)P(B)$
$\because$ A and B are independent events.
$\Rightarrow P(A' \cap B') = P(A')\,P(B')$

16 Let E = The event that six occurs
and A = The event that the man reports that it is a six
$\therefore P\left(\dfrac{E}{A}\right) = \dfrac{P(E)P(A/E)}{P(E)P(A/E) + P(E')P(A/E')}$
$= \dfrac{(1/6)(3/4)}{(1/6)(3/4) + (5/6)(1/4)} = \dfrac{3}{8}$

17 Letters of the word STATISTICS are A, C, I, I, S, S, S, T, T, T. Letters of the word ASSISTANT are A, A, I, N, S, S, S, T, T. Common letters are A, I, S and T.

Probability of choosing A is

$$\frac{1}{10} \times \frac{2}{9} = \frac{2}{90}.$$

Probability of choosing I is

$$\frac{2}{10} \times \frac{1}{9} = \frac{2}{90}.$$

Probability of choosing S is

$$\frac{3}{10} \times \frac{3}{9} = \frac{9}{90}.$$

Probability of choosing T is

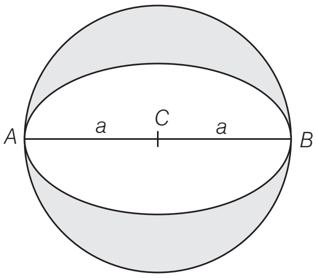
$$\frac{3}{10} \times \frac{2}{9} = \frac{6}{90}.$$

$\therefore$ Probability of required event

$$= \frac{2}{90} + \frac{2}{90} + \frac{9}{90} + \frac{6}{90} = \frac{19}{90}$$

18 Let equation of ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ (where, } a > b)$$



$\therefore$ Area of an ellipse $= \pi ab$

$$= \pi a \times a\sqrt{1 - e^2}$$

$$= \pi a^2 \sqrt{1 - \frac{8}{9}} = \frac{\pi a^2}{3}$$

and area of circle $= \pi a^2$

Now, required probability

$$= \frac{\pi a^2 - \dfrac{\pi a^2}{3}}{\pi a^2} = \frac{2}{3}$$

19 The total number of four-digit numbers formed with the digits $1, 2, \dots, 8$ is $^8C_4 \times 4!$.

Now, the total number of four-digits numbers formed with the digits 1, 2, …, 8 and containing unity as one of the digits is $^7C_3 \times 4!$.

$\therefore$ Required probability

$$= \frac{^7C_3 \times 4!}{^8C_4 \times 4!} = \frac{1}{2}$$

20 Let the mean of the last four digit be A_2.

Then, $\quad 12.5 = \dfrac{6 \times 15 + 4 \times A_2}{6 + 4}$

$\Rightarrow \quad 125 = 90 + 4A_2$

$\Rightarrow \quad A_2 = \dfrac{35}{4}$

Let the sixth number be x, then

$$10 = \frac{1 \times x + 4 \times \dfrac{35}{4}}{1 + 4}$$

$\Rightarrow \quad 50 = x + 35$

$\Rightarrow \quad x = 15$

21 $\text{GM} = (7 \cdot 7^2 \dots 7^n)^{1/n}$

$$= \left(7^{\frac{n(n+1)}{2}}\right)^{1/n} = 7^{\frac{n+1}{2}}$$

22 Clearly any month, out of 12 months, can be chosen with probability $= \dfrac{1}{12}$

Now, as there are 7 possible ways in which the month can start and it will be a Saturday on 14th day if the first day of the month is Sunday.

$\therefore$ Its probability $= \dfrac{1}{7}$

Hence, the required probability

$$= \frac{1}{12} \times \frac{1}{7} = \frac{1}{84}.$$

23 Consider

$$(1 + x)^{30} = {}^{30}C_0 + {}^{30}C_1 x + {}^{30}C_2 x^2 + \dots + {}^{30}C_{30} x^{30} \quad \dots\text{(i)}$$

and $(1 - x)^{30} = {}^{30}C_0 - {}^{30}C_1 x + {}^{30}C_2 x^2 - \dots + {}^{30}C_{30} x^{30} \quad \dots$(ii)

On adding Eqs. (i) and (ii), we get

$${}^{30}C_0 + {}^{30}C_2 x^2 + \dots + {}^{30}C_{30} x^{30} = \frac{1}{2}[(1 + x)^{30} + (1 - x)^{30}]$$

Now, on integrating over 0 to 1, we get

$$\left[{}^{30}C_0 x + \frac{{}^{30}C_2 x^3}{3} + \dots + \frac{{}^{30}C_{30} x^{31}}{31}\right]_0^1 = \frac{1}{2}\left[\frac{(1 + x)^{31}}{31} - \frac{(1 - x)^{31}}{31}\right]_0^1$$

$$\Rightarrow \frac{{}^{30}C_0}{1} + \frac{{}^{30}C_2}{3} + \dots + \frac{{}^{30}C_{30}}{31} = \frac{1}{2 \times 31}[2^{31}] = \frac{2^{30}}{31}$$

Now, as there are 16 terms, therefore required mean

$$= \frac{\dfrac{{}^{30}C_0}{1} + \dfrac{{}^{30}C_2}{3} + \dots + \dfrac{{}^{30}C_{30}}{31}}{16}$$

$$= \frac{\dfrac{2^{30}}{31}}{16} = \frac{2^{30}}{31 \times 16} = \frac{2^{26}}{31}$$

24 Since, $\quad 0 \le \sqrt{npq} < \sqrt{np}$

$\Rightarrow \quad 0 \le \text{SD} < 5 \quad [\because p \ne 0]$

$\Rightarrow \quad \text{SD} \in [0, 5)$

25 Here, $\sim p$: She is not intelligent.

$\sim q$: She is not studious.

So, the required symbolic form is $\sim(\sim p \vee \sim q)$ or $p \wedge q$.

26 Clearly,

$$p \to (\sim p \vee q) \equiv (\sim p) \vee ((\sim p) \vee q)$$

$$\equiv ((\sim p) \vee (\sim p)) \vee q \quad \text{[by associative law]}$$

$$\equiv (\sim p) \vee q \text{ [by idempotent law]}$$

Now, $\sim(p \to (\sim p \vee q)) \equiv \sim((\sim p) \vee q)$

$$\equiv \sim(\sim p) \wedge (\sim q)$$

$$\equiv p \wedge \sim q$$

27 Using Distributive law,

$$(p \vee r) \wedge (q \vee r) \equiv (p \wedge q) \vee r$$

28 Clearly, the roots of a quadratic equation can be imaginary. So, this statement has truth value F.

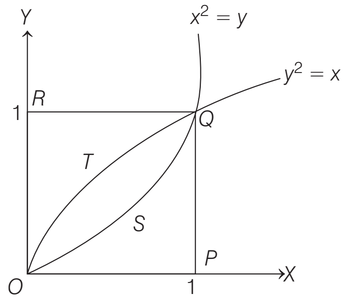
29 Here, $\quad u = \dfrac{x}{h} - \dfrac{a}{h}$

We know that standard deviation is not dependent on change of origin.

$\therefore \quad \sigma_u = \dfrac{\sigma_x}{h} \Rightarrow \sigma_x = h\sigma_u$

30 A = The event of (x, y) belonging to the area $OTQPO$

and B = The event of (x, y) belonging to the area $OSQRO$



$$P(A) = \frac{\text{ar}(OTQPO)}{\text{ar}(OPQRO)} = \frac{\int_0^1 \sqrt{x}\, dx}{1 \times 1}$$

$$= \left[\frac{2}{3} x^{3/2}\right]_0^1 = \frac{2}{3}$$

$$P(B) = \frac{\text{ar}(OSQRO)}{\text{ar}(OPQRO)} = \frac{\int_0^1 \sqrt{y}\, dy}{1 \times 1} = \frac{2}{3}$$

$$P(A \cap B) = \frac{\text{ar}(OTQS)}{\text{ar}(OPQRO)}$$

$$= \frac{\int_0^1 \sqrt{x}\, dx - \int_0^1 x^2 dx}{1 \times 1} = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

Note (i) $P(A) + P(B) = \dfrac{2}{3} + \dfrac{2}{3} \ne 1$

So, A and B are not exhaustive.

(ii) $P(A) \cdot P(B) = \dfrac{2}{3} \cdot \dfrac{2}{3} = \dfrac{4}{9} \ne P(A \cap B)$

So, A and B are not independent.

(iii) $P(A \cap B) \ne 0$, so A and B are not mutually exclusive.

31 Given, $P(A \cap B) = \frac{1}{8}$

and $P(\bar{A} \cap \bar{B}) = \frac{1}{4}$

$\therefore \quad P(A) \cdot P(B) = \frac{1}{8}$

and $P(\overline{A \cup B}) = \frac{1}{4}$

$\Rightarrow \quad P(A) \cdot P(B) = \frac{1}{8}$

and $P(A \cup B) = 1 - \frac{1}{4} = \frac{3}{4}$

$\Rightarrow \quad \frac{3}{4} = P(A) + P(B) - \frac{1}{8}$

$[\because P(A \cup B) = P(A) + P(B) - P(A \cap B)]$

$\Rightarrow \quad P(A) + P(B) = \frac{7}{8}$

Let $P(A) = x$, then $P(B) = \frac{1}{8x}$

$\therefore \quad x + \frac{1}{8x} = \frac{7}{8}$

$\Rightarrow \quad \frac{8x^2 + 1}{8x} = \frac{7}{8}$

$\Rightarrow \quad 8x^2 + 1 = 7x$

or $\quad 8x^2 - 7x + 1 = 0$

$\therefore \quad x = \frac{7 \pm \sqrt{49 - 32}}{16}$

$P(A) = \frac{7 \pm \sqrt{17}}{16}$

$\Rightarrow \quad P(B) = \frac{2}{7 \pm \sqrt{17}}$

32 We know, $p \Rightarrow q$ is false when p is true and q is false.

$\therefore p \Rightarrow (q \vee r)$ is false when p is true and $(q \vee r)$ is false, and we know $q \vee r$ is false only when both q and r are false.

Hence, truth values of p, q, and r are respectively T, F, F.

33 Clearly, number of workers in shift A

$= \frac{3}{6} \times 1500 = 750$

Number of workers in shift B

$= \frac{2}{6} \times 1500 = 500$

and number of workers in shift $C = 250$

Now, let the sum of wages per person in three shifts $= x$, then

Wages in shift $A = \frac{4}{15} \times x$

$\Rightarrow \quad \frac{4}{15} \times x = 400$

$\Rightarrow \quad x = 1500$

Now, wages in shift B

$= \frac{5}{15} \times 1500 = 500$ per person

and wages in shift C

$= \frac{6}{15} \times 1500 = 600$ per person

$\therefore$ Mean wage

$\frac{750 \times 400 + 500 \times 500 + 250 \times 600}{1500}$

$=$ ₹ 467 per worker

34 Let the three students be A, B and C.

Also, let E, F and G denote the events of standing first of three students A, B and C respectively. Then, we have

$P(E) = \frac{1}{1+2} = \frac{1}{3}; P(F) = \frac{2}{2+5} = \frac{2}{7}$

and $P(G) = \frac{1}{1+7} = \frac{1}{8}$.

Since, the events E, F and G are mutually exclusive.

$\therefore P(E \cup F \cup G) = P(E) + P(F) + P(G)$

$= \frac{1}{3} + \frac{2}{7} + \frac{1}{8}$

$= \frac{56 + 48 + 21}{168} = \frac{125}{168}$

35 Since, n is odd, therefore $^{2n}C_0, {}^{2n}C_1, {}^{2n}C_2, \ldots, {}^{2n}C_n$ are even in number.

Now,

$$\text{median} = \frac{1}{2}\left[\left(\frac{n+1}{2}\right)^{\text{th}} \text{observation} + \left(\frac{n+1}{2} + 1\right)^{\text{th}} \text{observation}\right]$$

$$= \frac{1}{2}\left[{}^{2n}C_{\frac{n+1}{2} - 1} + {}^{2n}C_{\frac{n+1}{2} + 1 - 1}\right]$$

$$= \frac{1}{2}\left[{}^{2n}C_{\frac{n-1}{2}} + {}^{2n}C_{\frac{n+1}{2}}\right]$$

36 Contrapositive of $(p \vee q) \to r$ is

$\sim r \to \sim (p \vee q) \equiv \sim r \to (\sim p \wedge \sim q)$

37 Since, $A = (x, x + 2, x + 4)$

and $\quad B = (x - 2, x + 2, x + 6)$

$\therefore$ Mean of A

$= \frac{x + x + 2 + x + 4}{3} = x + 2$

and mean of

$B = \frac{x - 2 + x + 2 + x + 6}{3} = x + 2$

Hence, group B has more variability than group A.

[$\because$ From the given data difference in scores of group A is 2 but difference in scores of group B is 4.]

38 Clearly, Statement I is true but Statement II is false, as negation of given statement is $p \Rightarrow q$.

39 $P(A \cup \bar{B}) = 1 - P(\overline{A \cup \bar{B}})$

$\Rightarrow \quad 0.8 = 1 - P(\bar{A} \cap B)$

$\Rightarrow \quad 0.8 = 1 - P(\bar{A})\, P(B)$

$\Rightarrow \quad 0.8 = 1 - 0.7 \times P(B)$

$\therefore \quad P(B) = \frac{2}{7}$

40

p	q	$\sim p$	$p \vee q$	$(p \vee q) \wedge \sim p$	$\sim p \wedge q$
T	T	F	T	F	F
T	F	F	T	F	F
F	T	T	T	T	T
F	F	T	F	F	F

It is clear from the table both statements are true and Statement II is a correct explanation for Statement I.